Executive Summary of the Project

TYPE

Minor Research Project (XII Plan of UGC)

TITLE

Analysis of Prime Generating Algorithms and Prime Distribution Patterns

UGC APPROVAL LETTER NO. AND DATE

File No: 47-748/13(WRO) Dated : 20 March 2014

PRINCIPAL INVESTIGATOR

Dr.N.A.Pande

DESIGNATION

Assistant Professor in Mathematics

AFFILIATION

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PROJECT OUTLAY

Rs. 1,70,000/- (Rs. One Lakh Seventy Thousand Only)

EXPENDITURE INCURRED

Rs. 1,70,585/- (Rs. One Lakh Seventy Thousand Five Hunderd and Eighty Five Only)

BOOKS AND JOURNALS

Sr. No.	Title of the Book	Author(s)	Publisher	ISBN(s)	Price in Rs.
1.	Numbers Galore: Perfects, Primes, Triples and Twins	Jack McCabe	Outskirts Press, Inc	978-1-4787-1767-4	1,321.00

Sr. No.	Title of the Book	Author(s)	Publisher	ISBN(s)	Price in Rs.
2.	Number Theory: Proofs of Flt, Cramers' Conjecture, Infinitude of Prime Twin Pairs and Goldbachs' Conjecture	Jere Housworth		978-1-5177-7326-7	995.00
3.	Prime Obsession: Berhhard Riemann and the Greatest Unsolved Problem in Mathematics	John Derbyshire	Penguin Books Ltd.	978-0-452-28525-5	914.00
4.	Visions of Infinity: The Great Mathematical Problems	Ian Stewart	Basic Books	978-0-465-06489-2	866.00
5.	Prime Numbers' Characteristics: Why They Are What They Are	L. J. Balasundaram		146378869X 978-1-463-78869-8	814.00
6.	The Prime Numbers And Their Distribution	Gerald Tenenbaum, Michel Mendes France	American Mathematical Society	978-1-4704-1917-2	608.00
7.	Prime Curios!: The Dictionary of Prime Number Trivia	Chris K. Caldwell, G.L.Honaker. Jr	CreateSpace	1-448-65170-0 978-1-448-65170-2	1,132.00
8.	The Theory of Prime Number Classification	Zwide Mbulawa	Xlibris Corporation	978-1-4535-9982-2	1,980.00
9.	Prime Numbers and Computer Methods for Factorization (Progress in Mathematics)	Hans Riesel	Springer	978-1-4612-6681-5	7,162.00
10.	Number Theory: An Introduction via the Distribution of Primes	Benjamin Fine, Gerhard Rosenberger	Birkhäuser	0-8176-4472-5 978-0-8176-4472-7	4,663.00

Sr. No.	Title of the Book	Author(s)	Publisher	ISBN(s)	Price in Rs.
11.	Primality Testing in Polynomial Time: From Randomized Algorithms to "PRIMES Is in P" (Lecture Notes in Computer Science)	Martin Dietzfelbinger	Springer	3-540-40344-2 978-3-540-40344-9	3,171.00
12.	Introduction to Modern Prime Number Theory (Cambridge Tracts in Mathematics and Mathematical Physics)	T. Esterman	Cambridge University Press	978-0-521-16828-1	1,338.00
13.	The Little Book of Bigger Primes	Paula Ribenboim	Springer	81-8128-270-1 979-8-1812-8270-4	344.00

BOOKS BEING GIVEN TO COLLEGE LIBRARY

All the 13 books purchased under this project have been deposited in the central library of the college have those have been give accession numbers 102106 to 102118.

DATE OF IMPLEMENTATION

02/05/2014

TENURE

2 (Two) Years

DATE OF FINALIZATION

April 2016

OBJECTIVE OF THE PROJECT

This Project started off with three main objectives :

1. Testing rigorously prime generating algorithms along with their newer and better versions evolved with time for identification of better performing candidate amongst these.

2. Generating sufficiently huge database of primes.

3. Using this database to study the distribution patterns of primes.

WORK DONE

As per the objectives of the project, initially deterministic algorithms for generation of primes were tested for superiority. Using rigorous comparison results, better algorithm (with impression of Eratosthenes) was chosen, and upon it a further modified one was designed. This allowed achieving the second objective of generation of huge database of prime numbers. Using this database, the distribution patterns of prime numbers were studied in arithmetical progressions $3n + k$, $4n + k$, $5n + k$ and $6n + k$, giving due respect to Dirichlet's Theorem, from various wide angles as mentioned in results achieved.

RESULTS ACHIEVED

Amongst as many as 18 prime generating algorithms selected for comparison studies, the best one could be identified; in fact, it has been built upon the famous sieve of Eratosthenes. It has proven that how simple modifications in classical techniques can bring in good progress for both speed of execution of algorithms and number of steps required. Prime database as huge as till 1,000,000,000,000 was generated; of course it has demanded long execution period! Finally, as per third objective, distribution patterns of primes were determined, particularly in arithmetical progressions, $3n + k$, $4n + k$, $5n + k$ and $6n + k$, wherein it is found that Dirichlet's Theorem undoubtedly holds very good; the arithmetical progression $3n + 2$ contain more primes than $3n + 1$ in most of the occasions; similarly, arithmetical progression $4n + 3$ contain more primes than $4n + 1$; and amongst family of $5n + k$ progressions, $5n + 2$ has taken marginal led in our final ranges; prime density in $6n + 5$ is frequently more than that in $6n + 1$. All these are trends observed commonly, but keeping their reputation of irregularity(!), the majority of number of primes often keeps

switching in these groups. Additionally, many other peculiar facts about prime numbers in the ranges of $1 - 10^n$ have been determined. This goes from very first range $1 - 10$ to as high range as $1 - 10^{12}$. In each of this range, first prime number of specific pattern is determined, when it existed. The last prime number is also traced out. Exhaustively searching all blocks of concerned sizes till 1 trillion, first and last of blocks with minimum number of primes and similarly first and last of blocks with maximum number of primes have been found out. The minimum and maximum prime density in such blocks is also determined. Further the block-wise smallest and largest gaps between successive primes of corresponding forms $an + b$ have been determined alongwith first and last prime starter of those gaps. Availability of count of primes of different forms with various digits occurring at units and tens places is also is another achievement of this project. All this rigorous analysis for all selected four forms is done systematically will surely help in future for finding some regularity, though complex, in occurrence of primes. All these details are made publicly available by as many as 9 publications in open access journals.

CONTRIBUTION TO THE SOCIETY

The outcomes of this project have many applications and will contribute to the advancement of society through them. This project has sorted out as many as 18 different algorithms for generations of primes; all of which are deterministic rather than probabilistic. The last few are built upon famous and classical Sieve of Eratosthenes and have improve this sieve through simple technique. Whenever there is a need to generate either a single large prime or the sequence of all primes till a specific value, the best of these algorithms is going to come very handy. As another outcome, huge database of prime numbers has been generated and is ready to be used for further work. Finally, the distribution patterns of primes in various arithmetic progressions had flashed light on prime density in all of those in huge ranges. The most important use of these all to the society is that better primes for

encryption can be chosen making internet communication, particularly online financial transactions, more secure.

PAPERS PUBLISHED OUT OF PROJECT WORK

Nine (9) papers have been published out of the project work :

UPLOADING EXECUTIVE SUMMARY OF THE REPORT

The Executive Summary of the report is uploaded to the college website.

FINAL COMMENT ABOUT THE PROJECT

The project has been completed as per plan and expectation of the Principal Investigator.

(Dr.N.A.Pande)   I/C Principal
Principal Investigator   (Seal)